there is the possibility of obtaining in this way nomograms representing equations of a form not easily represented by other methods.

OTHER CURVES AS INDICES

In general, to construct similar nomograms with curved indices first select as an index a curve which is determined by a certain number of points, say n. This index should be such that it can be constructed without excessive difficulty when n points are arbitrarily given. The corresponding nomogram will be one for an equation in n + 1 variables. We select origins and supports for n + 1 scales. The nature of the index curve serves as an important factor in this selection. The equation represented by the nomogram is then found by a consideration of the equation of the index with the equation of the supports.

THE CIRCLE AS AN INDEX

Three points will determine a circle

$$A(x^2 + y^2) + Bx + Cy + D = 0$$

This would be an accurate and practical index since it is easy to construct a circle through three points. It would give us a nomogram for an equation in four variables. A necessary and sufficient condition that four points (x_1, y_1), (x_2, y_2), (x_3, y_3), and (x_4, y_4) lie upon a circle is:

$$\begin{vmatrix} x^2_1 + y^2_1 & x_1 & y_1 & 1 \\ x^2_2 + y^2_2 & x_2 & y_2 & 1 \\ x^2_3 + y^2_3 & x_3 & y_3 & 1 \\ x^2_4 + y^2_4 & x_4 & y_4 & 1 \end{vmatrix} = 0$$

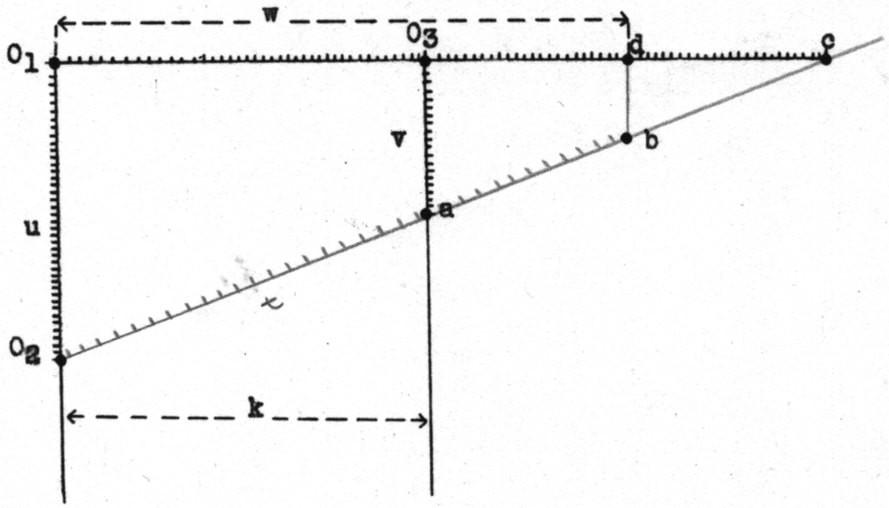
By substituting functions for the values x_ν, y_ν we get the general equation representable by a nomogram with a circle as index.

LABELED INDICES

The labeled index provides us with another tool for representing an additional variable. To obtain an alignment nomogram for four variables we set up three labeled curves and a labeled index line in such a way that three of the variables may be chosen at will and the fourth is then determined. We find the equation represented by a consideration of the geometric nature of the curves.

As one scheme of this sort choose O_1 as origin for the u scale (see figure, pp 8) on one vertical straight line and also as the origin of the w scale to be carried upon a perpendicular straight line extending to the right. Choose O_3 as the origin of the v scale which is to have a support parallel to that of the u scale and at a distance k from it. We shall require that the origin of the labeled index *(also a straight line) shall always be the point determined when the value of u is given. We then connect the index and the w scale by a line perpendicular to the latter and the nomogram is complete. Similar nomograms can be formed by choosing other slopes for the line bd or by describing a point through which it must pass.

The equation represented by this particular nomogram may be found as follows:



By similar triangles $\frac{t}{w} = \frac{\overline{O_2c}}{\overline{O_1c}}$, $\frac{v}{u} = \frac{\overline{O_3c}}{\overline{O_1c}}$

Therefore $\overline{O_2c}\,\frac{w}{t} = \overline{O_3c}\,\frac{u}{v}$

But $\overline{O_2c} = \frac{u\sqrt{k^2 + (u - v)^2}}{u - v}$, $\overline{O_3c} = \frac{vk}{u - v}$

Therefore $w\sqrt{k^2 + (u - v)^2} = tk$

* (Indices and lines auxiliary to indices are drawn in red pencil)

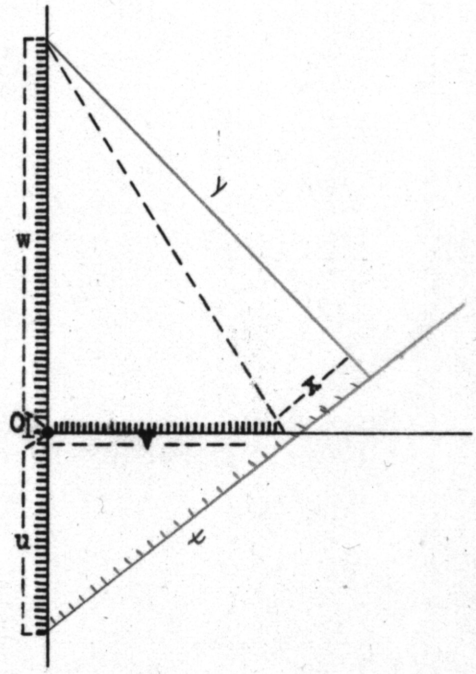
If $u = f_1$, $v = f_2$, $w = f_3$, and $t = f_4$ where each f is a function of some variable we get:

$$f_3\sqrt{k^2 + (f_1 - f_2)^2} = kf_4$$

By taking logarithms we notice that this equation is representable by a combination of simple nomograms. This combination, however, is likely to be more confusing in its application than the labeled index nomogram.

Consider a second scheme for the representation of a four variable equation by use of the labeled index line. Upon a straight line choose O_1 as the common origin of the w scale ascending (see figure, pp 9), of the u scale descending, and of the v scale which is carried upon a perpendicular support extending to the right. The labeled index shall have its origin at the point determined by the value of u and shall intersect the v scale at the point determined by the value of v. The index line shall carry the scale t. From the point determined by the value of t we draw a perpendicular to the index which will intersect the w scale. Any three of these variables are then independent and together they determine a value for the fourth.

The equation is found as shown:



In the right triangles $x^2 + y^2 = w^2 + v^2$

$$(u + w)^2 - t^2 = y^2$$

$$u^2 + v^2 = (t - x)^2$$

Therefore $w^2 + v^2 = (t \pm \sqrt{u^2 + v^2})^2 + (u + w)^2 - t^2$

(We choose the plus sign here for otherwise $x > t$)

Therefore $u(u + w) = t\sqrt{u^2 + v^2}$

If $u = f_1$, $v = f_2$, $w = f_3$, and $t = f_4$ we get:

$$f_1(f_1 + f_3) = f_4 \sqrt{f^2{}_1 + f^2{}_2}$$

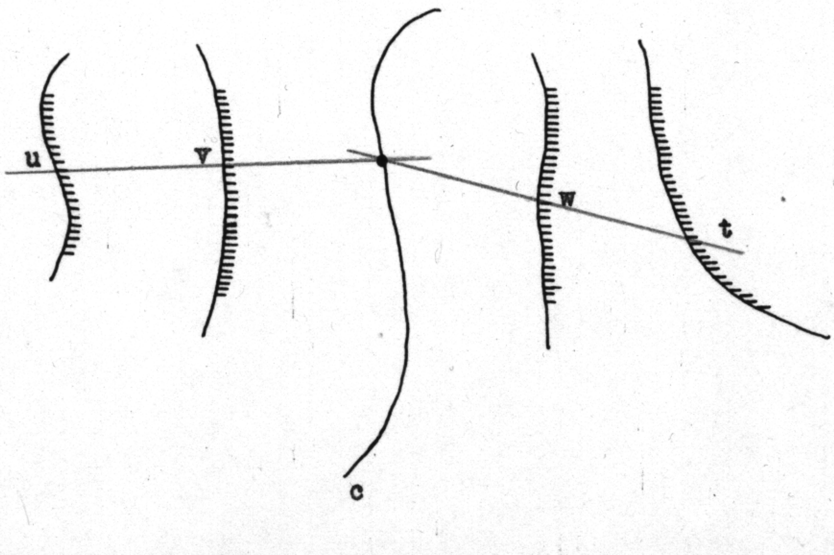
COMBINATION OF TWO STRAIGHT LINE INDICES

By making use of two straight line indices it is possible to devise nomograms for equations involving four variables. Various special combinations have been studied in detail but there seems to be no attempt in the literature to group these special cases into one theory.

Assume that we have an index consisting of two straight lines and that the values of the four variables u, v, w, and t are carried upon four curved supports. When three of the variables are known the fourth must be determined.

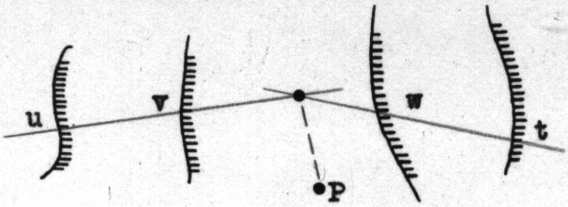
The position of a straight line may be established either by two points or by a point and a slope. When values of any two of the variables, say u and v, are given, two points are determined upon the supports. These two points will fix the position of one index line. Now the point determined by the value of the third variable together with some condition which we may assign will locate the second index line. The assigned condition (a point or a slope) must depend in some way upon the values of u and v if an equation is to connect the four variables.

If the condition is a point, we may choose the intersection of the first index with any fixed curve C.

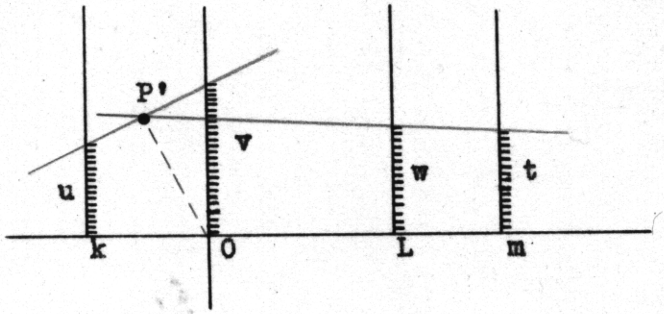


In this case we recognize the so called double alignment chart. The theory of such charts has been worked out by studying the two charts of which it is composed. Charts which do not result from this consideration can be devised by simply choosing methods of combining two indices.

For example, let the first index be determined by the values of u and v. Let P be any fixed point. Let the second index be determined by the value of the third given variable, say t, and by the intersection of the first index and the perpendicular to it through P.



As a particular case take supports all parallel to the y axis, the origins all on the x axis, and P as (0, 0):

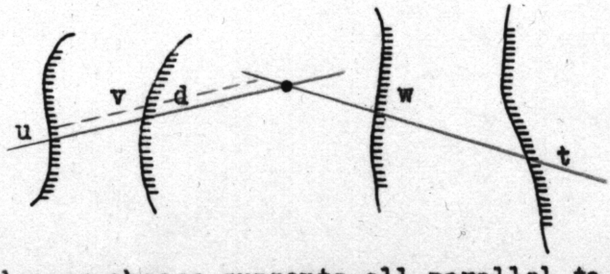


The points marked u, v, w, and t are then (k, u), (0, v), (L, w), and (m, t) respectively. The co-ordinates of P' may then be found and we write the equation of the line through P' and the point (m, t). In this equation we replace x and y by L and w and we have as the equation represented by this nomogram

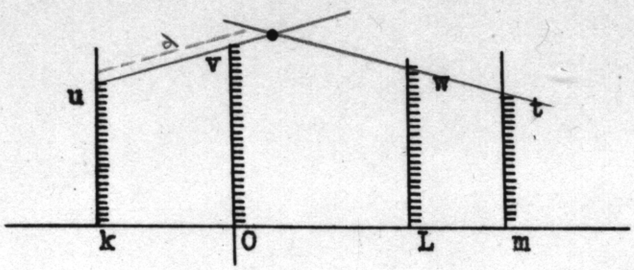
$$w - t = \frac{\left[k^2(v - t) - t(u - v)^2\right] \ (L - m)}{(v - u)\left[um + v(k - m)\right] - mk^2}$$

By introducing functional scales we can, of course, replace the four variables by functions.

We may assign the condition that the second index be determined by the value of the third given variable, say t, and another point P to be determined as follows: From the point of intersection of the first index and the u support lay off on the first index a constant distance d toward the v support. Then let the termination point of d on the first index be the point P.



As a special case choose supports all parallel to the y axis and origins all on the x axis.



The equation thus represented is

$$w - t = \frac{\left[d(u - v) + (u - t)\sqrt{(u - v)^2 + k^2}\right](L - m)}{k\left[d + \sqrt{(u - v)^2 + k^2}\,(k - m)\right]}$$

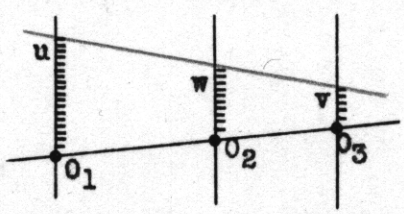
If we wish the assigned condition to be a slope we may choose any slope with respect to the first index. The case of parallel and perpendicular indices has been extensively treated. Two indices having any given slope with respect to each other may be made parallel or perpendicular by a suitable rotation transformation of two supports.

Of course we could combine any number of straight lines as indices by repeating one of the above methods for each new index. Curved indices may also be combined by analogous procedures.

COMMON TYPES OF EQUATIONS

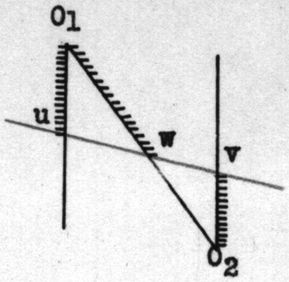
As a rule, equations for which a nomogram is desired fall into one of the following groups:

(1) $f_1 + f_2 + f_3 + \ldots\ldots\ldots\ldots = 0$ In the case of three functions of three variables this type may be represented by a nomogram consisting of three parallel supports with collinear origins and a straight index.

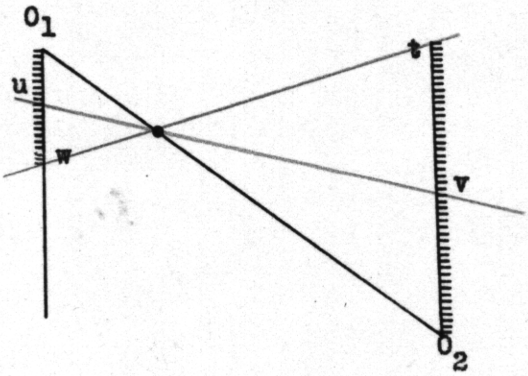


Equations of similar form in larger numbers of variables may be represented by a combination of such charts.

(2) $f_1 = f_2 f_3$ A simple chart for equations of this type consists of two parallel and one transversal support where O_1 is the origin of the u and w scales and O_2 the origin of the v scale. The index is straight.

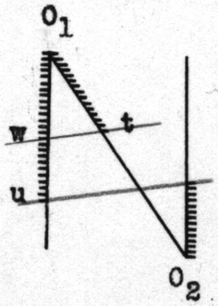


(3) $\frac{f_1}{f_2} = \frac{f_3}{f_4}$ may be represented by two parallel supports each carrying two scales and having origins on a transversal. The index is two straight lines intersecting on the transversal.

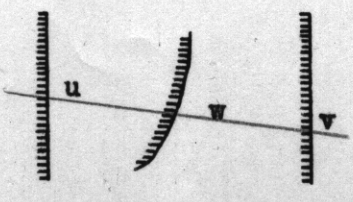


Combining such charts we can represent f = f f f' .....

(4) $f_1 + f_2 = \frac{f_3}{f_4}$ This may be charted by two parallel and one transversal support with two parallel straight lines as indices.



(5) $f_1(u) + f_2(v) \cdot f_3(w) = f_4(w)$ The nomogram for the equations of this type consists of two parallel straight and one curved support. The index is a straight line.



There are, of course, alternate methods for representing each of these five types of equations.

If an equation cannot be put into one of the above forms it can usually be reduced to several equations each of which is chartable by one of the given methods. By combining the nomograms for the several equations we have a representation for the original equation.

REMARKS

Often the position of a support depends upon the value of some constant. By making this support movable we may introduce a new variable in place of the constant and thus obtain a nomogram for an equation involving one more variable.

By making use of the principle of duality it is possible to replace "point" by "line" and "line" by "point" in any theorem involving lines and points and the relationship "on". This is a valuable tool in nomography.

The consideration of nomograms in space has, so far, received little attention. Such nomograms offer material for extensive theoretical work. Perhaps one reason why such work has not been done is the fact that such a large percentage of the equations found in practical fields are chartable by simple known methods.